

Phases of Brans-Dicke Cosmology(II):Matter from NS-NS sector

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Abstract

We study the cosmology of the Brans-Dicke(BD) theory coupled to perfect fluid type matter. In our previous works, the case where matter is coming from the Ramond-Ramond sector of the string theory was studied. Here we study the case where matter is coming from NS-NS sector. Exact solutions are found and the cosmology is classified according to the values of γ , the parameter of the equation of state and ω , the BD parameter. We find that, in string frame, there are solutions without singularity for some ranges of γ and ω . In Einstein frame, however, all solutions are singular.

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I. INTRODUCTION

The string theory is believed to be the most promising candidate to quantum gravity. So it is natural to expect that it will resolve some problems inherent in the general relativity like the initial singularity problem. In fact, in the regime of Planck length curvature, quantum fluctuation is very large so that string coupling becomes large and consequently the fundamental string degrees of freedom are not weakly coupled *good* ones. Instead, solitonic degrees of freedom like solitonic p-branes [1] or D-brane [2] are more important. Therefore it is an interesting question to ask whether including these degrees of freedom resolve the initial singularity.

The new gravity theory that can deal with such new degree of freedom should be a deformation of standard general relativity so that in a certain limit it should be reduced to the standard Einstein theory. The Brans-Dicke theory [3] is a generic deformation of the general relativity allowing variable gravity coupling. Therefore whatever is the motivation to modify the Einstein theory, the Brans-Dicke theory is the first one to be considered. As an example, low energy limit of the string theory contains the Brans-Dicke theory with a fine tuned deformation parameter ($\omega = -1$) and it is extensively studied under the name of the string cosmology [4–6]. Without knowing the exact theory of the p-brane cosmology, the best guess is that it should be a Brans-Dicke theory with matters. In fact there is some evidence for this [1], where it is found that the natural metric that couples to the p-brane is the Einstein metric multiplied by certain power of dilaton field. In terms of this new metric, the action that gives the p-brane solution becomes Brans-Dicke action with definite deformation parameter ω depending on p.

In our previous works [7,8], we studied the gas of solitonic p-brane [1] treated as a perfect fluid type matter in a Brans-Dicke theory allowing the equation of state parameter γ arbitrary. We had studied the case where the perfect fluid does not couple to the dilaton like the matter in the Ramond-Ramond sector of the string theory. Here we study the opposite case where matter couples to the dilaton like those coming from NS-NS sector (see reference

[9] for similar study). Exact solutions are found both in string and Einstein frame and the cosmology is classified according to the values of γ and ω . In string frame we will find non-singular solutions for some ranges of γ and ω . In Einstein frame, however, we will find that all solutions are singular, unlike the string frame.

The rest of this paper is organized as follows. In section II, we construct the action for the case when the matter coupled to the dilaton and find analytic solutions for the equations motion. In section III, the relation between cosmic time t and parameter τ is studied. In section IV, we consider the behavior of the scale factor a as a function of τ . Using these results, in section V, we study the scale factor as a function of the cosmic time t . In section VI, we study the asymptotic behavior of $a(t)$ and classify them according to acceleration and deceleration phases. Up to the section VI, all analyses were done in string frame. In section VII, we investigate the cosmology in Einstein frame. In section VIII, we summarize and conclude with some discussion.

II. ACTION WITH SOLITONIC NS-NS MATTER AND ITS SOLUTIONS

We begin with the four dimensional Brans-Dicke like string action of which matter is coupled to the dilaton field.

$$S = \int d^4x \sqrt{-g} e^{-\phi} [R - \omega(\nabla\phi)^2 + L_m] \quad (1)$$

Notice that the action differs from that in reference [7,8] by the coupling of the matter Lagrangian L_m with the dilaton factor. By varying the action, we get equations of motion:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= T_{\mu\nu} + \omega\{\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\} \\ &+ \{-\nabla_\mu\nabla_\nu\phi + \nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\nabla^2\phi - g_{\mu\nu}(\nabla\phi)^2\} \\ R - 2\omega\nabla^2\phi + \omega(\nabla\phi)^2 + L_m &= 0. \end{aligned} \quad (2)$$

Let's choose the metric as

$$ds^2 = -Ndt^2 + e^{2\alpha(t)}dx_idx^i \quad (i = 1, 2, 3),$$

where N is a lapse function. We consider perfect fluid type matter whose energy-momentum tensor is

$$T_{\mu\nu} = pg_{\mu\nu} + (p + \rho)U_\mu U_\nu,$$

satisfying the conservation law

$$\dot{\rho} + 3(p + \rho)\dot{\alpha} = 0.$$

Using the equation of state, $p = \gamma\rho$, we get

$$\rho = \rho_0 e^{-3(1+\gamma)\alpha}.$$

So, we can rewrite the action as

$$S = \int dt e^{3\alpha-\phi} \left[\frac{1}{\sqrt{N}} \{-6\dot{\alpha}^2 + 6\dot{\alpha}\dot{\phi} + \omega\dot{\phi}^2\} - \sqrt{N}\rho_0 e^{-3(1+\gamma)\alpha} \right]. \quad (3)$$

Now we define new variable τ by

$$d\tau e^{3\alpha-\phi} = dt. \quad (4)$$

Then, the action becomes

$$\begin{aligned} S &= \int d\tau \left[\frac{1}{\sqrt{N}} \{-6\alpha'^2 + 6\alpha'\phi' + \omega\phi'^2\} - \sqrt{N}\rho_0 e^{3(1-\gamma)\alpha-2\phi} \right], \\ &= \int d\tau [\Gamma_1 Y'^2 + \Gamma_3 X'^2 - \rho_0 e^{-2X}], \end{aligned} \quad (5)$$

where

$$\begin{aligned} \Gamma_1 &= -6 + 9(1-\gamma) + \frac{9}{4}(1-\gamma)^2\omega = \frac{9}{4}(1-\gamma)^2(\omega - \omega_{\Gamma_1}), \\ \Gamma_2 &= 6 + 3(1-\gamma)\omega = 3(1-\gamma)(\omega - \omega_{\Gamma_2}), \\ \Gamma_3 &= \omega - \frac{\Gamma_2^2}{4\Gamma_1} = -\frac{3(2\omega+3)}{\Gamma_1}, \end{aligned} \quad (6)$$

$$-2X = 3(1-\gamma)\alpha - 2\phi,$$

$$Y = \alpha + \frac{\Gamma_2}{2\Gamma_1}X, \quad (7)$$

$$\begin{aligned}\omega_{\Gamma_1} &= \frac{4(3\gamma - 1)}{3(1 - \gamma)^2}, \\ \omega_{\Gamma_2} &= -\frac{2}{1 - \gamma} = \omega_\eta.\end{aligned}\tag{8}$$

From this action we get equations of motion:

$$\begin{aligned}Y'' &= 0, \\ X'' - \frac{\rho_0}{\Gamma_3} e^{-2X} &= 0.\end{aligned}\tag{9}$$

The constraint equation is obtained by varying lapse function N :

$$\Gamma_1 Y'^2 + \Gamma_3 X'^2 + \rho_0 e^{-2X} = 0.\tag{10}$$

The behavior of the solution depend crucially on the sign of Γ_1 .

- $\Gamma_1 < 0$ case:

$$\begin{aligned}X &= \ln \left[\frac{q}{c} \cosh c\tau \right], \\ Y &= A\tau + B.\end{aligned}\tag{11}$$

where, c, A, B and $q = \sqrt{\frac{\rho_0}{|\Gamma_3|}}$ are arbitrary real constants. Using the cosntraint equation, we can determine A in terms of other variables

$$A = c \sqrt{\frac{-\Gamma_3}{\Gamma_1}} = c \frac{\sqrt{3(2\omega + 3)}}{|\Gamma_1|}.\tag{12}$$

- $\Gamma_1 > 0$ case:

$$\begin{aligned}X &= \ln \left[\frac{q}{c} |\sinh c\tau| \right], \\ Y &= A\tau + B.\end{aligned}\tag{13}$$

Having been found X and Y , α and ϕ can be found from the relation Eq.(7).

In next section, we will find $t(\tau)$ and find that its behavior depends on ω and γ .

III. PHASE SPACE CLASSIFICATION IN TERMS OF T AND τ

A. $\Gamma_1 < 0$ case

From Eqs.(4) and (11), $t(\tau)$ is found to be

$$t - t_0 = \int d\tau e^{\frac{3}{2}(1+\gamma)(A\tau+B)} \left[\frac{q}{c} \cosh(c\tau) \right]^{-\frac{3\Gamma_2}{4\Gamma_1}(1+\gamma)-1}. \quad (14)$$

It is easy to see $t(\tau)$ is a monotonic function. When τ goes to $\pm\infty$, t can be approximately integrated to be

$$t \sim \frac{1}{T_{\pm}} e^{T_{\pm}\tau}, \quad (15)$$

where

$$T_{\pm} = -\frac{3\sqrt{3(2\omega+3)}}{2\Gamma_1}(1+\gamma) \mp \left[\frac{3\Gamma_2}{4\Gamma_1}(1+\gamma) + 1 \right]. \quad (16)$$

We have fixed $c = 1$. We will say that t is supermonotonic function of τ when it is monotonic and t runs the entire real line when τ does. When t is supermonotonic function of τ , the universe evolves from infinite past to infinite future. Otherwise, the universe has a starting(ending) point at a finite cosmic time $t_i(t_f)$. The running range of t depend on the sign of T .

$$\begin{aligned} -\infty < t < \infty & \text{ if } T_- < 0 < T_+, \\ -\infty < t < t_f & \text{ if } T_- < 0 \text{ and } T_+ < 0, \\ t_i < t < \infty & \text{ if } T_- > 0 \text{ and } T_+ > 0, \\ t_i < t < t_f & \text{ if } T_+ < 0 < T_-. \end{aligned} \quad (17)$$

The solution for $T_- < 0$ is found to be

$$-\frac{3}{2} < \omega < -\frac{4}{3}, \text{ and } \omega > \omega_{\kappa}, \quad (18)$$

where we have defined

$$\frac{(3\gamma-5)}{3(1-\gamma)} := \omega_{\kappa}.$$

To get Eq.(18) we used following identity.

$$3(2\omega + 3) = \left(\frac{\Gamma_2}{2}\right)^2 - \omega\Gamma_1.$$

The region II in Fig.1 is correspond to this solution. For $T_- > 0$, we have solution:

$$\omega > -\frac{3}{2}, \quad \omega < \omega_\kappa, \quad \text{or} \quad \omega > -\frac{4}{3}. \quad (19)$$

The region I and VII is satisfy this solution.

If $T_+ < 0$, the solution to Eq.(16) is

$$-\frac{3}{2} < \omega < -\frac{4}{3}, \quad \text{and} \quad \omega < \omega_\kappa. \quad (20)$$

The region I correspond to this solution. For $T_+ > 0$, we have

$$\omega > -\frac{3}{2}, \quad \omega > \omega_\kappa, \quad \text{or} \quad \omega > -\frac{4}{3}. \quad (21)$$

The region II and VII satisfy this solution.

B. $\Gamma_1 > 0$ case

In this case the solution $X(\tau)$ in Eq.(13) has a singularity at $\tau = 0$. So we have to look at the behavior of t near $\tau = 0$ carefully. From Eqs.(4) and (13), the $t(\tau)$ can be written as

$$t - t_0 = \int d\tau e^{\frac{3}{2}(1+\gamma)[\frac{\sqrt{3(2\omega+3)}}{\Gamma_1}c\tau+B]} \left[\frac{q}{c} |\sinh(c\tau)| \right]^{-\frac{3\Gamma_2}{4\Gamma_1}(1+\gamma)-1}. \quad (22)$$

The asymptotic behavior of t in the limit $\tau \rightarrow \pm\infty$, is given by

$$t \sim \frac{1}{T_\pm} e^{T_\pm \tau}, \quad (23)$$

where

$$T_\pm = \frac{3\sqrt{3(2\omega+3)}}{2\Gamma_1}(1+\gamma) \mp \left[\frac{3\Gamma_2}{4\Gamma_1}(1+\gamma) + 1 \right]. \quad (24)$$

Notice the difference from Eq.(15).

The condition $T_- < 0$ gives solution:

$$\omega > -\frac{3}{2}, \quad \omega < \omega_\kappa, \quad \text{and} \quad \omega > -\frac{4}{3}. \quad (25)$$

There is no region satisfying this solution. For $T_- > 0$ the solution is

$$\omega > -\frac{3}{2}, \quad \omega > \omega_\kappa, \quad \text{or} \quad \omega < -\frac{4}{3}. \quad (26)$$

Therefore III, IV, V and VI satisfies this solution.

The solution for $T_+ < 0$ is

$$\omega > -\frac{3}{2}, \quad \omega > \omega_\kappa, \quad \text{and} \quad \omega > -\frac{4}{3}. \quad (27)$$

The region V and VI satisfy this solution. For $T_+ > 0$ the solution is

$$\omega > -\frac{3}{2}, \quad \omega < \omega_\kappa, \quad \text{or} \quad \omega < -\frac{4}{3}. \quad (28)$$

The region III and IV satisfy this solution.

So far our analysis is parallel to the previous section. However, we have to pay attention to the behavior of $t(\tau)$ near $\tau = 0$. In the limit $\tau \rightarrow 0$,

$$t \sim \frac{\text{sign}(\tau)}{1-\eta} |\tau|^{1-\eta} \quad (29)$$

where $\eta = \frac{3\Gamma_2}{4\Gamma_1}(1+\gamma) + 1$. Notice $t(\tau)$ is regular at $\tau = 0$, if $\eta < 1$. When $\eta > 1$, $t(\tau)$ is singular at $\tau = 0$. So we consider $t(\tau)$ in the region $-\infty < \tau < 0$ and $0 < \tau < \infty$ separately.

The condition $\eta > 1$ is equivalent to

$$\omega > \omega_{\Gamma_2}. \quad (30)$$

When τ goes to zero from the below,

$$t \sim \frac{(-\tau)^{1-\eta}}{\eta-1}, \quad (31)$$

which means $t \rightarrow \infty$ as $\tau \rightarrow -0$. On the other hand, when τ goes to zero from the above,

$$t \sim -\frac{(-\tau)^{1-\eta}}{\eta-1}, \quad (32)$$

so that $t \rightarrow -\infty$ as $\tau \rightarrow +0$.

From these analysis, we see that the parameter space of γ and ω is divided into seven regions as depicted in Fig.1.

FIGURES

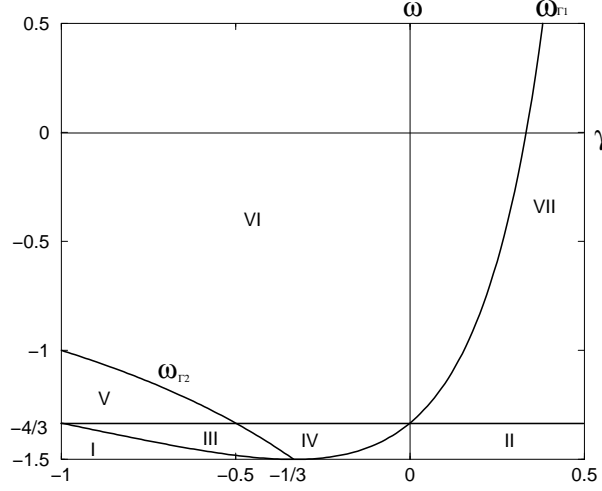


FIG. 1. The phases classified by the relation between t and τ .

We summarize the results that are found in this section.

- Region I: $T_+ < 0$, $T_- > 0$, $\Gamma_1 < 0$; t evolves from initial time t_i to final time t_f for $\tau \in (-\infty, \infty)$.
- Region II: $T_- < 0$, $T_+ > 0$, $\Gamma_1 < 0$; t evolves from negative infinity to positive infinity for $\tau \in (-\infty, \infty)$.
- Region III: $T_+ > 0$, $T_- > 0$, $\Gamma_1 > 0$; t evolves from initial time t_i to positive infinity for $\tau \in (-\infty, \infty)$.
- Region IV: $T_- > 0$, $T_+ > 0$, $\Gamma_1 > 0$; Since $\tau = 0$ is singular, the region of τ is divided into two regions. t evolves from initial time t_i to positive infinity for $\tau \in (-\infty, 0)$; t evolves from negative infinity to positive infinity for $\tau \in (0, \infty)$.
- Region V: $T_+ < 0$, $T_- > 0$, $\Gamma_1 > 0$; t evolves from initial time t_i to final time t_f for $\tau \in (-\infty, \infty)$.
- Region VI: $T_+ < 0$, $T_- > 0$, $\Gamma_1 > 0$; Since t is singular at $\tau = 0$, we should divide into two regions. t evolves from initial time t_i to positive infinity for $\tau \in (-\infty, 0)$ and negative infinity to final time t_f for $\tau \in (0, \infty)$.

- Region VII: $T_- > 0$, $T_+ > 0$, $\Gamma_1 < 0$; t evolves from initial time t_i to positive infinity for $\tau \in (-\infty, \infty)$.

IV. THE BEHAVIOR OF THE SCALE FACTOR

We now study the phases of the cosmology by looking at the scale factor $a(\tau) = \exp(\alpha(\tau))$.

A. $\Gamma_1 < 0$ case

In this case $\alpha(\tau)$ in scale factor $e^{\alpha(\tau)}$ is given by

$$\alpha(\tau) = \frac{c\sqrt{3(2\omega+3)}}{-\Gamma_1}\tau + B - \frac{\Gamma_2}{2\Gamma_1} \left[\ln \frac{q}{c} \cosh(c\tau) \right]. \quad (33)$$

In the limit $\tau \rightarrow \pm\infty$, the scale factor can be written as

$$a(\tau) \sim e^{H_{\pm}\tau}, \quad (34)$$

where the H_{\pm} is defined by

$$H_{\pm} = -\frac{c\sqrt{3(2\omega+3)}}{\Gamma_1} \mp \frac{\Gamma_2}{2\Gamma_1}c. \quad (35)$$

Eq.(35) for $H_- < 0$ gives the solution

$$\omega > \omega_{\Gamma_2}, \text{ and } \omega < 0. \quad (36)$$

The region II in Fig.2 satisfies this solution. For $H_- > 0$ the solution is

$$\omega < \omega_{\Gamma_2}, \text{ or } \omega > 0. \quad (37)$$

The region I and VI satisfy this solution.

If $H_+ < 0$, the solution is given by

$$\omega < \omega_{\Gamma_2}, \text{ and } \omega < 0. \quad (38)$$

The region I satisfies this solution. For $H_+ > 0$ the solution is

$$\omega > \omega_{\Gamma_2}, \text{ or } \omega > 0. \quad (39)$$

The region II and VI satisfy this solution.

B. $\Gamma_1 > 0$ case

In this case, the $\alpha(\tau)$ in scale factor $e^{\alpha(\tau)}$ is given by

$$\alpha(\tau) = \frac{c\sqrt{3(2\omega+3)}}{\Gamma_1}\tau + B - \frac{\Gamma_2}{2\Gamma_1} \left[\ln \frac{q}{c} |\sinh(c\tau)| \right]. \quad (40)$$

In the limit $\tau \rightarrow \pm\infty$, $a(\tau)$ is given by

$$a(\tau) \sim e^{H_{\pm}\tau}, \quad (41)$$

where H_{\pm} is defined by

$$H_{\pm} = \frac{c\sqrt{3(2\omega+3)}}{\Gamma_1} \mp \frac{\Gamma_2}{2\Gamma_1}c. \quad (42)$$

The solution to the condition $H_- < 0$ is

$$\omega < \omega_{\Gamma_2}, \text{ and } \omega > 0. \quad (43)$$

There is no region satisfying this solution. For $H_- > 0$ the solution is

$$\omega > \omega_{\Gamma_2}, \text{ or } \omega < 0. \quad (44)$$

The satisfying region is III, IV and V.

The solution for $H_+ < 0$ case is

$$\omega > \omega_{\Gamma_2}, \text{ and } \omega > 0. \quad (45)$$

The region V satisfies this solution. For $H_+ > 0$ the solution is given by

$$\omega < \omega_{\Gamma_2}, \text{ or } \omega < 0. \quad (46)$$

The region III and IV satisfy this solution.

Now we consider $\tau \rightarrow 0$ limit. In this limit $a(\tau)$ is approximately given by

$$a(\tau) \sim |\tau|^{-\frac{3(1-\gamma)(\omega-\omega_{\Gamma_2})}{2\Gamma_1}}. \quad (47)$$

Therefore, if $\omega > \omega_{\Gamma_2}$, $a(\tau)$ goes to infinite as $\tau \rightarrow 0$. From these analyses we get a phase diagram Fig.2.

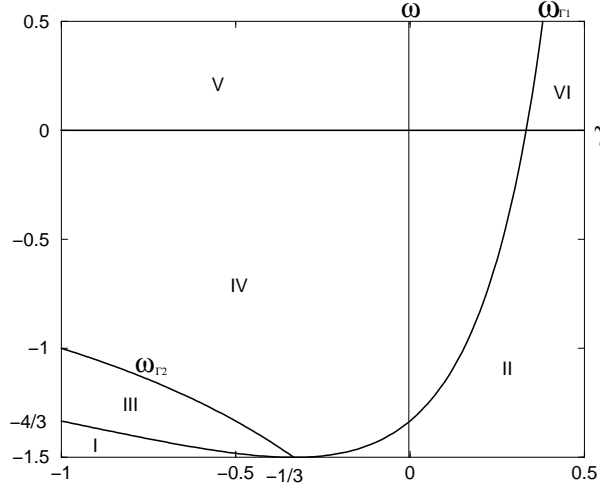


FIG. 2. The parameter space is classified by the scale factor $a(\tau)$

Summarizingly, we have following cases.

- Region I: $H_- > 0$, $H_+ < 0$, $\Gamma_1 < 0$; The scale factor $a(\tau)$ goes to zero size as $\tau \rightarrow \pm\infty$.
- Region II: $H_- < 0$, $H_+ > 0$, $\Gamma_1 < 0$; $a(\tau)$ goes to infinity as $\tau \rightarrow \pm\infty$.
- Region III: $H_- > 0$, $H_+ > 0$, $\Gamma_1 > 0$; In this region, the behavior of t is not singular, so we need not consider the behavior of $a(\tau)$ at $\tau = 0$ where the scale factor vanishes. $a(\tau)$ goes to zero as $\tau \rightarrow -\infty$ and $a(\tau)$ goes to infinity as $\tau \rightarrow \infty$.
- Region IV: $H_- > 0$, $H_+ > 0$, $\Gamma_1 > 0$; The $a(\tau)$ goes to infinite size as τ goes to zero for $\tau \in (-\infty, 0)$ and zero size as $\tau \rightarrow -\infty$. $a(\tau)$ goes to infinity as τ goes to zero for $\tau \in (0, \infty)$ and goes to positive infinity as $\tau \rightarrow \infty$.

- Region V: $H_- > 0$, $H_+ < 0$, $\Gamma_1 > 0$; $a(\tau)$ goes to infinity as τ goes to zero and $a(\tau)$ goes to zero as τ goes to negative infinity for $\tau \in (-\infty, 0)$. $a(\tau)$ goes to infinity as τ goes to zero and a goes to zero τ infinity for $\tau \in (0, \infty)$.
- Region VI: $H_- > 0$, $H_+ > 0$, $\Gamma_1 < 0$; In the limit of $\tau \rightarrow -\infty$, $a(\tau)$ goes to zero size. In the limit of $\tau \rightarrow \infty$, $a(\tau)$ goes to infinite size.

V. PHASES OF THE COSMOLOGY

In previous sections we have studied $t(\tau)$ and $a(\tau)$. From all considerations of these results, we can classify the parameter space of γ and ω by the behavior of $a(t)$ into sixteen phases. In Fig.3, we show the phase diagram. In asymptotic region where $\tau \rightarrow \pm\infty$, we can write the scale factor $a(t)$ as:

$$\begin{aligned} a(t) &\sim [T_-(t - t_i)]^{\frac{H_-}{T_-}} \\ a(t) &\sim [T_+(t - t_f)]^{\frac{H_+}{T_+}}. \end{aligned} \quad (48)$$

From these relations we see that the behavior of the scale factor $a(t)$ depends on the sign of T_{\pm} and the value of H_{\pm}/T_{\pm} determines acceleration or deceleration of the scale factor which is discussed below. In Fig.4 - Fig.6, we show the behavior of scale factor by numerical study. The $-$ sign in $V-$ indicates the branch for $\tau \in (-\infty, 0)$. Similarly, $V+$ means $\tau \in (0, \infty)$.

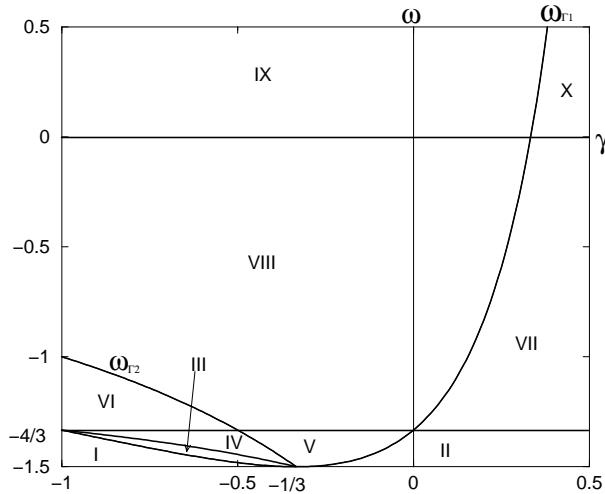
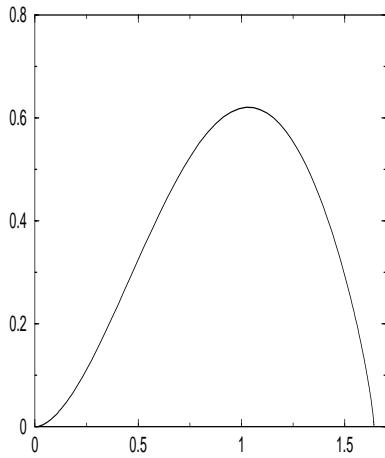


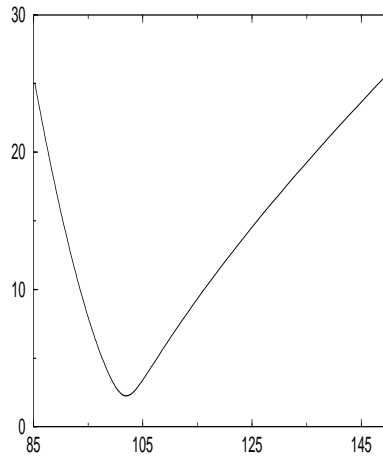
FIG. 3. The parameter space is classified by the behavior of $a(t)$

- Region I, $T_- > 0, T_+ < 0, H_- > 0$ and $H_+ < 0$. The universe evolves from zero size at a finite initial time t_i to a zero size at a finite final time t_f . This region contains the matter for inflation $\gamma = -1$. However, we see in the limit $\tau \rightarrow -\infty$ the cosmic time t approaches to t_i .
- Region II, $T_- < 0, T_+ > 0, H_- < 0$ and $H_+ > 0$. The universe evolves from infinite to infinite size as t runs from negative infinity to positive infinity. This is the region where we can find the non-singular behavior of $a(t)$.
- Region III, $T_+ > 0, T_- > 0, H_- > 0$ and $H_+ > 0$. The universe evolves from zero size to infinite as t runs from finite initial time t_i to infinity for $\tau \in (-\infty, \infty)$. During evolution the universe goes to zero as τ goes to zero. So we divided into two branches at $\tau = 0$.
- Region IV, $T_- > 0, T_+ > 0, H_- > 0$ and $H_+ > 0$. The universe evolves from zero to infinite size as t runs from finite t_i to infinity. Like region *III* we divided into two branches since the universe becomes zero as τ goes to zero.

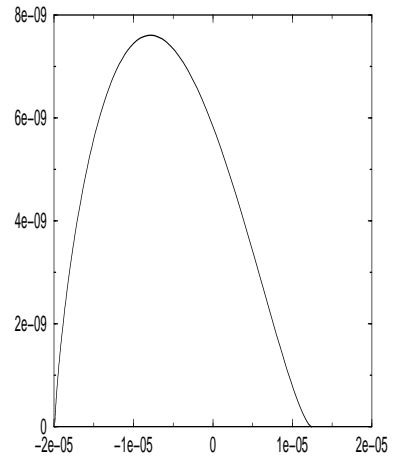
By numerical study we depicted the behavior of scale factor $a(t)$.



(a) phase I



(b) phase II



(c) phase III-

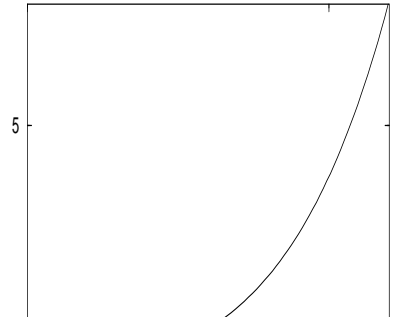
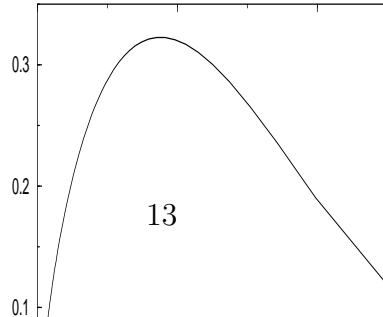
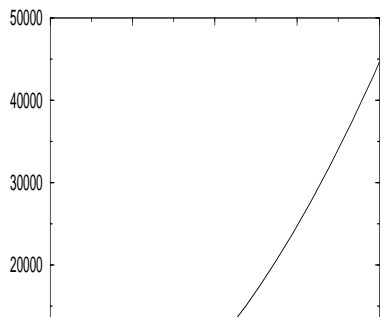


FIG. 4. The behavior of the scale factor from phase *I* to *IV*

- Region V, $T_- > 0, T_+ > 0, H_- > 0$ and $H_+ > 0$. The universe evolves from zero to infinite size as t runs finite initial time t_i to infinity and the universe evolves from infinite to infinite as t runs negative infinity to infinity.
- Region VI, $T_- > 0, T_+ < 0, H_- > 0$ and $H_+ > 0$. The universe evolves from zero to zero size as t runs from initial time t_i to infinity and the universe evolves from zero to infinite size as t runs negative infinity to finite final time t_f .
- Region VII, $T_- > 0, T_+ > 0, H_- < 0$ and $H_+ > 0$. The universe evolves from infinite to infinite size as t runs from finite initial time t_i to infinity.

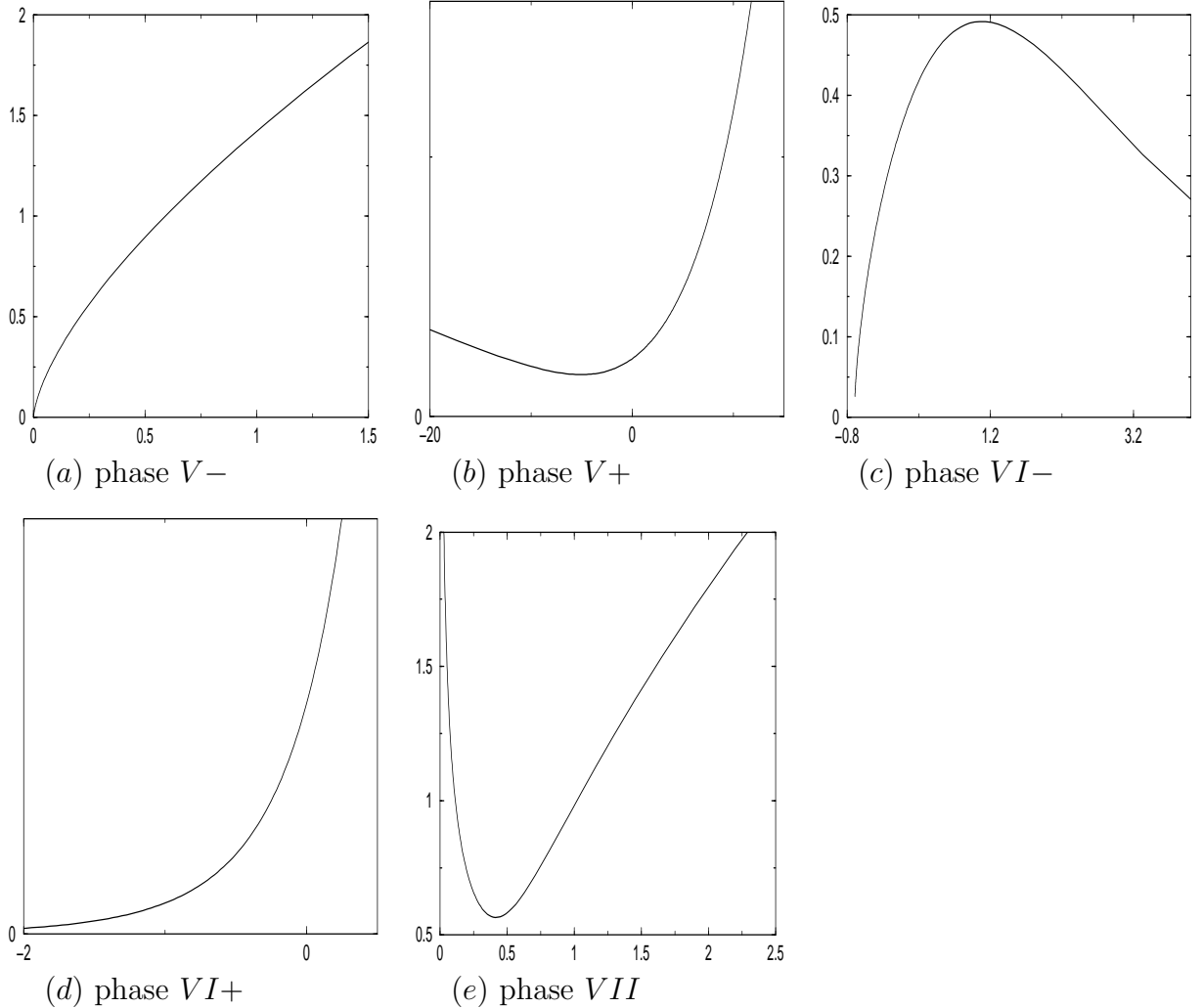


FIG. 5. The behavior of the scale factor from phase *V* to *VII*

- Region VIII, $T_- > 0, T_+ < 0, H_- > 0$ and $H_+ > 0$. The universe evolves from zero to infinite size as t runs from finite initial time t_i to infinity and the universe evolves from infinite to infinite size as t runs negative infinity to finite final time t_f .
- Region IX, $T_- > 0, T_+ < 0, H_- > 0$ and $H_+ < 0$. The universe evolves from zero to infinite size as t runs from finite initial time t_i to infity and the universe evolves from infinite size to zero as t runs from negative infinity to finite final time t_f .
- Region X, $T_- > 0, T_+ > 0, H_- > 0$ and $H_+ > 0$. The universe evolves from zero to infinite size as t runs from finite initial time t_i to infinity.

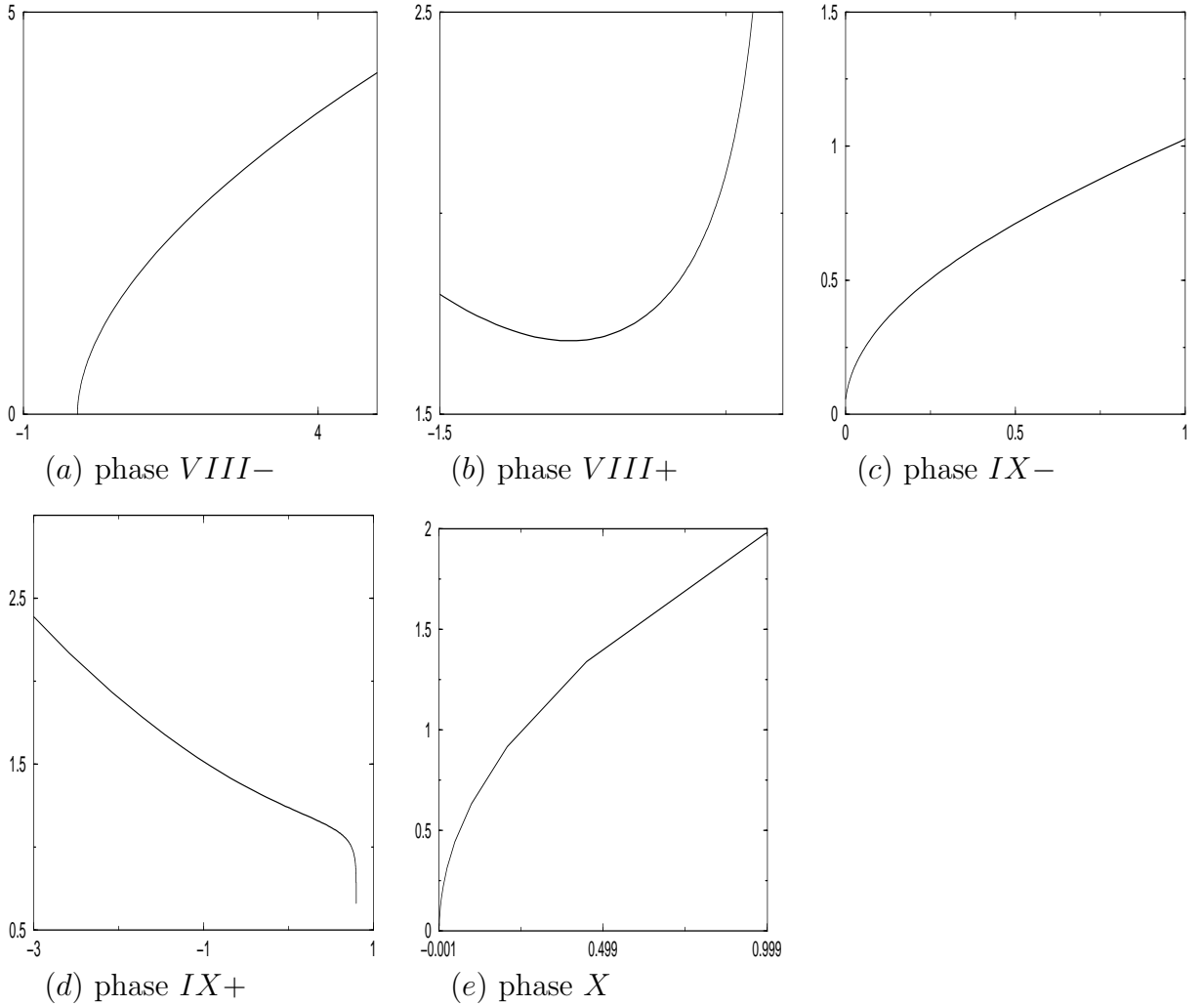


FIG. 6. The behavior of the scale factor from phase *VIII* to *X*

VI. ACCELERATION / DECELERATION PHASE

Notice that as we have seen in Eq.(49) not only the sign of H_{\pm}/T_{\pm} but also that of $H_{\pm}/T_{\pm} - 1$ is important because the universe will accelerate or decelerate according to the sign of the latter.

A. $\Gamma_1 < 0$ case

1. $H_-/T_- > 1$

For $T_- > 0$, the condition $H_-/T_- > 1$ is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} < 3(1 - \gamma)(2\omega + 3). \quad (49)$$

Consider $\gamma > -\frac{1}{3}$ case first. To our surprise, the inequality (49) gives us $\omega > \frac{4(3\gamma-1)}{3(1-\gamma)^2} = \omega_{\Gamma_1}$, namely $\Gamma_1 > 0$. This is contradiction. Now for $\gamma < -\frac{1}{3}$, Eq.(49) gives $\omega < \omega_{\Gamma_1}$. The region I in figure 3 corresponds to this case.

For $T_- < 0$, the $H_-/T_- > 1$ is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} > 3(1 - \gamma)(2\omega + 3). \quad (50)$$

This inequality gives $\omega < \omega_{\Gamma_1}$ for $\gamma > -\frac{1}{3}$. The region II corresponds to this. For $\gamma < -1/3$, we get the condition $\omega > \omega_{\Gamma_1}$ which contradicts to $\Gamma_1 < 0$. In a summary, region I and II satisfies $H_-/T_- > 1$, $\Gamma_1 < 0$.

2. $H_+/T_+ > 1$

For $T_+ > 0$, the condition $H_+/T_+ > 1$ is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} < -3(1 - \gamma)(2\omega + 3). \quad (51)$$

The analysis is completely similar to the case A. For $\gamma < -1/3$, from above inequality we get $\omega < \omega_{\Gamma_1}$. Since $T_+ > 0$ is satisfied only by the region II, III, IV, VII, it is easy to

see that there is no region satisfying all three conditions, $\Gamma_1 < 0, T_+ > 0, \gamma < -1/3$. For $\gamma > -1/3$ Eq.(51) gives $\omega > \omega_{\Gamma_1}$ which contradicts to $\Gamma_1 < 0$.

For $T_+ < 0, H_+/T_+ > 1$ is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} > -3(1 - \gamma)(2\omega + 3). \quad (52)$$

When $\gamma > -1/3$, the above inequality gives $\omega < \omega_{\Gamma_1}$. Since the condition $T_+ < 0$ is only satisfied by the region I, V, VI, there is no region satisfying three conditions $\Gamma_1 < 0, T_+ < 0, \gamma > -1/3$. When $\gamma < -1/3$, Eq.(51) gives $\gamma > \omega_{\Gamma_1}$ which contradicts to $\Gamma_1 < 0$. In a summary, it is always $H_+/T_+ < 1, \Gamma_1 < 0$.

B. $\Gamma_1 > 0$ case

1. $H_-/T_- > 1$

For $T_- > 0$, the condition $H_-/T_- > 1$ is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} < -3(1 - \gamma)(2\omega + 3). \quad (53)$$

Consider $\gamma > -1/3$ case first. The left hand side is always positive while the right hand side is always negative. So there is no solution for this. For $\gamma < -1/3$ case. Eq.(53) gives $\omega < \omega_{\Gamma_1}$ which contradicts to $\Gamma_1 > 0$.

For $T_- < 0$ case, from section III, we know that there is no region satisfying $T_- < 0$ condition. Therefore there are no solutions for conditions $\Gamma_1 > 0, H_-/T_- > 1$. We summarize if $\Gamma_1 > 0$ then we have $H_-/T_- < 1$ for all region.

2. $H_+/T_+ > 1$

For $T_+ > 0, H_+/T_+ > 1$ is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} < 3(1 - \gamma)(2\omega + 3). \quad (54)$$

Consider $\gamma > -1/3$ case. The above inequality gives $\omega > \omega_{\Gamma_1}$. Part of the region V satisfies these conditions. For the solution to $\gamma < -1/3$ case, Eq.(54) gives $\omega < \omega_{\Gamma_1}$ which contradicts to $\Gamma_1 > 0$.

For $T_+ < 0$, $H_+/T_+ > 1$ is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} > 3(1 - \gamma)(2\omega + 3). \quad (55)$$

When $\gamma < -1/3$, we find solution $\omega > \omega_{\Gamma_1}$ which is satisfied by the region VI in Fig 3. For $\gamma > -1/3$ case, Eq.(55) gives $\omega < \omega_{\Gamma_1}$ which contradicts $\Gamma_1 > 0$.

We summarize what we have obtained so far by a table.

phase	sign of Γ_1	sign of T_-	sign of T_+	range of t	H_-/T_-	H_+/T_+
I	−	+	−	$[t_i, t_f]$	$H_-/T_- > 1$	$0 < H_+/T_+ < 1$
II	−	−	+	$(-\infty, \infty)$	$H_-/T_- > 1$	$0 < H_+/T_+ < 1$
III [−]	+	+		$[t_i, t_f]$	$0 < H_-/T_- < 1$	$H_+/T_+ > 1$
III ⁺	+		+	$[t_i, \infty)$		
IV [−]	+	+		$[t_i, t_f]$	$0 < H_-/T_- < 1$	$H_+/T_+ > 1$
IV ⁺	+		+	$[t_i, \infty)$		
V [−]	+	+		$[t_i, \infty)$	$0 < H_-/T_- < 1$	$H_+/T_+ > 1$
V ⁺	+		+	$(-\infty, \infty)$		
VI [−]	+	+		$[t_i, t_f]$	$0 < H_-/T_- < 1$	$H_+/T_+ < 0$
VI ⁺	+		−	$[t_i, \infty)$		
VII	−	+	+	$[t_i, \infty)$	$H_-/T_- < 0$	$0 < H_+/T_+ < 1$
VIII [−]	+	+		$[t_i, \infty)$	$0 < H_-/T_- < 1$	$H_+/T_+ < 0$
VIII ⁺	+		−	$(-\infty, t_f]$		
IX [−]	+	+		$[t_i, \infty)$	$0 < H_-/T_- < 1$	$0 < H_+/T_+ < 1$
IX ⁺	+		−	$(-\infty, t_f]$		
X	−	+	+	$[t_i, \infty)$	$0 < H_-/T_- < 1$	$0 < H_+/T_+ < 1$

VII. EINSTEIN FRAME

In this section, we study the cosmology in Einstein frame. Especially, we investigate the difference between the behavior of the scale factor in Einstein frame and that in the string frame as well as the possibility to avoid the initial singularity.

The metric in Einstein frame is obtained from string frame metric by transformation using the relation $g_{E\mu\nu} = e^{-\phi} g_{\mu\nu}$:

$$\begin{aligned} ds_E^2 &= e^{-\phi} ds^2 \\ &= -e^{-\phi} dt^2 + e^{2\alpha-\phi} dx_i dx^i \\ &= -dt_E^2 + e^{2\alpha_E} dx_i dx^i \quad (i = 1, 2, 3). \end{aligned} \quad (56)$$

From above relations, we see $\alpha_E = \alpha - \frac{\phi}{2}$ and $dt_E = e^{-\frac{\phi}{2}} dt$. Then we can obtain solutions in Einstein frame combining the above relation with the solutions in string frame. Therefore the solutions in Einstein frame are

$$\alpha_E(\tau) = c \frac{(3\gamma + 1)\sqrt{3(2\omega + 3)}}{-4\Gamma_1} \tau - \ln \left[\frac{q}{c} \cosh(c\tau) \right] \left(\frac{(3\gamma + 1)\Gamma_2 + 4\Gamma_1}{8\Gamma_1} \right) + \frac{3\gamma + 1}{4} B, \quad (57)$$

for $\Gamma_1 < 0$, and

$$\alpha_E(\tau) = c \frac{(3\gamma + 1)\sqrt{3(2\omega + 3)}}{4\Gamma_1} \tau - \ln \left[\frac{q}{c} |\sinh(c\tau)| \right] \left(\frac{(3\gamma + 1)\Gamma_2 + 4\Gamma_1}{8\Gamma_1} \right) + \frac{3\gamma + 1}{4} B, \quad (58)$$

for $\Gamma_1 > 0$. In next section we find out $t_E(\tau)$ and see that the interval of $t_E(\tau)$ can be classified by ω and γ .

A. Classification of the phases by t_E and τ

1. $\Gamma_1 < 0$ case

From Eq.(4) in section II, and using Eqs.(56) and (57), $t_E(\tau)$ can be read

$$\begin{aligned} t_E - t_{E0} &= \int d\tau e^{3\alpha_E(\tau)} \\ &\sim \int d\tau e^{\frac{3(3\gamma+1)\sqrt{3(2\omega+3)}}{-4\Gamma_1} c\tau} [\cosh(c\tau)]^{-\frac{3[4\Gamma_1+(3\gamma+1)\Gamma_2]}{8\Gamma_1}}. \end{aligned} \quad (59)$$

In the limit $\tau \rightarrow \pm\infty$, we can write

$$t_E \sim \frac{1}{T_{\pm}} e^{T_{E\pm}\tau}.$$

where

$$T_{E\pm} = -\frac{3(3\gamma+1)\sqrt{3(2\omega+3)}}{4\Gamma_1} \mp \frac{3[4\Gamma_1 + (3\gamma+1)\Gamma_2]}{8\Gamma_1}. \quad (60)$$

The condition for $T_{E-} < 0$ is reduced to

$$(3\gamma+1)\sqrt{3(2\omega+3)} < 3(1-\gamma)(2\omega+3). \quad (61)$$

Consider first $\gamma > -\frac{1}{3}$ case. The solution for Eq.(61) is $\omega > \omega_{go}$ which violate $\Gamma_1 < 0$. For $\gamma < -\frac{1}{3}$, we get $\omega < \omega_{\Gamma_1}$. Now $T_{E-} > 0$ case that is

$$(3\gamma+1)\sqrt{3(2\omega+3)} > 3(1-\gamma)(2\omega+3). \quad (62)$$

For $\gamma > -\frac{1}{3}$, Eq.(62) gives $\omega < \omega_{\Gamma_1}$. However, for $\gamma < -\frac{1}{3}$, the left hand side is negative while right hand side is positive which is inconsistent.

Now consider $T_{E+} < 0$ case which is reduced to

$$(3\gamma+1)\sqrt{3(2\omega+3)} < -3(1-\gamma)(2\omega+3). \quad (63)$$

For $\gamma > -\frac{1}{3}$, in Eq.(63) the left hand side is positive while right hand side negative which is inconsistent. For $\gamma < -\frac{1}{3}$, we have $\omega < \omega_{\Gamma_1}$. Consider T_{E+} which is reduced to

$$(3\gamma+1)\sqrt{3(2\omega+3)} > -3(1-\gamma)(2\omega+3). \quad (64)$$

For $\gamma > -\frac{1}{3}$, we have $\omega < \omega_{\Gamma_1}$. For $\gamma < -\frac{1}{3}$, Eq.(64) gives $\omega > \omega_{\Gamma_1}$ which violates $\Gamma_1 < 0$.

One can summarize that under the condition $\omega < \omega_{\Gamma_1}$,

$$\text{For } \gamma < -\frac{1}{3} : T_{E-} < 0, T_{E+} < 0.$$

$$\text{For } \gamma > -\frac{1}{3} : T_{E-} > 0, T_{E+} > 0.$$

These relations tell us that

$$\text{For } \gamma < -\frac{1}{3} : -\infty < t_E < t_{Ef}.$$

$$\text{For } \gamma > -\frac{1}{3} : t_{Ei} < t_E < \infty.$$

We emphasize that there is no region where t_E can run from $-\infty$ to $+\infty$. This is because T_{E+} and T_{E-} have the same sign in any given region unlike the string frame.

2. $\Gamma_1 > 0$ case

In this case $t_E(\tau)$ can be read from Eqs.(4), (56) and (58),

$$\begin{aligned} t_E - t_{E0} &= \int d\tau e^{3\alpha_E(\tau)} \\ &\sim \int d\tau e^{\frac{3(3\gamma+1)\sqrt{3(2\omega+3)}}{4\Gamma_1} c\tau} |\sinh(c\tau)|^{-\frac{3(4\Gamma_1+(3\gamma+1)\Gamma_2)}{8\Gamma_1}}. \end{aligned} \quad (65)$$

As we saw above, this case is singular as $\tau \rightarrow 0$. So it is necessary to consider the behavior in that case. As $\tau \rightarrow 0$,

$$t_E \sim \text{sign}(\tau) \frac{|\tau|^{1-\eta}}{1-\eta}, \quad (66)$$

where $\eta = \frac{3[(3\gamma+1)\Gamma_2+4\Gamma_1]}{8\Gamma_1}$. If $\eta > 1$, t_E is singular at $\tau = 0$, while if $\eta < 1$, it is regular. $\eta > 1$ case gives

$$\omega > -\frac{(5+3\gamma)}{3(1-\gamma^2)} = \omega_*$$

The other case, $\eta < 1$, gives $\omega < \omega_*$ which does not overlap with $\Gamma_1 > 0$. Therefore there is no regular region for $\Gamma_1 > 0$. As $\tau \rightarrow -\infty$, $t_E \rightarrow +\infty$ while as $\tau \rightarrow +0$, $t_E \rightarrow -\infty$. Let us find out the behavior in the region $\tau \rightarrow \pm\infty$. In this limit t_E and τ is given by

$$t_E - t_{E0} \sim \int d\tau e^{T_{E\pm}\tau}, \quad (67)$$

where

$$T_{E\pm} = \frac{3(3\gamma+1)\sqrt{3(2\omega+3)}}{4\Gamma_1} \mp \frac{3[(3\gamma+1)\Gamma_2+4\Gamma_1]}{8\Gamma_1}. \quad (68)$$

The condition $T_{E-} > 0$ is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} > -3(1 - \gamma)(2\omega + 3). \quad (69)$$

For $\gamma > -\frac{1}{3}$, the left hand side is always positive while the right hand side always negative. So Eq.(69) satisfies trivially. For $\gamma < -\frac{1}{3}$, Eq.(69) gives $\omega > \omega_{\Gamma_1}$ which is consistent with $\Gamma_1 > 0$.

For $T_{E-} < 0$, we have the inequality

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} < -3(1 - \gamma)(2\omega + 3). \quad (70)$$

For $\gamma > -\frac{1}{3}$, since after dividing by $3\gamma + 1$ the left hand side is always positive while the right hand side is always negative. So there is no solution. For $\gamma < -\frac{1}{3}$, Eq.(70) gives $\omega < \omega_{\Gamma_1}$ which contradicts to $\Gamma_1 > 0$.

Now consider the case $T_{E+} > 0$. This condition is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} > 3(1 - \gamma)(2\omega + 3). \quad (71)$$

Consider first $\gamma > -\frac{1}{3}$. Eq.(71) gives $\omega < \omega_{\Gamma_1}$ which contradicts to $\Gamma_1 > 0$. For $\gamma < -\frac{1}{3}$, the left hand side is always positive while the right hand side always is negative. So we have no solution for $T_{E+} > 0$.

Consider $T_{E+} < 0$ which is reduced to

$$(3\gamma + 1)\sqrt{3(2\omega + 3)} < 3(1 - \gamma)(2\omega + 3). \quad (72)$$

For $\gamma > -\frac{1}{3}$, Eq.(72) gives $\omega > \omega_{\Gamma_1}$ which satisfies $\Gamma_1 > 0$. For $\gamma < -\frac{1}{3}$, in Eq.(72) after dividing by $3\gamma + 1$ we see that the left hand side is positive while the right hand side is negative. It is easy to check that when $\omega > \omega_{\Gamma_1}$,

$$T_{E-} < 0, \text{ and } T_{E+} < 0.$$

In a summary, for $\Gamma_1 > 0$, t_E is singular as $\tau \rightarrow 0$. t_E runs from initial time t_{Ei} to $+\infty$ for $\tau \in (-\infty, 0)$ and $-\infty$ to final time t_{Ef} for $\tau \in (0, \infty)$.

B. The scale factor

In previous section we see that $t_E(\tau)$ is not monotonic function which is crucial to decide whether the scale factor is singular or not. In this section following the same procedure we study the behavior of the scale factor, $a_E(\tau)$, and classify by ω and γ .

1. $\Gamma_1 < 0$ case

For this case we use the Eq.(57), then

$$\begin{aligned} a_E(\tau) &= e^{\alpha_E(\tau)} \\ &= c_1 e^{\frac{(3\gamma+1)\sqrt{3(2\omega+3)}}{-4\Gamma_1}\tau} \left[\frac{q}{c} \cosh(c\tau) \right]^{-\frac{(3\gamma+1)\Gamma_2+4\Gamma_1}{8\Gamma_1}}, \end{aligned} \quad (73)$$

where $c_1 = e^{\frac{(3\gamma+1)B}{4}}$.

In the limit $\tau \rightarrow \pm\infty$, the scale factor can be rewritten as $e^{H_{E\pm}\tau}$ where

$$H_{E\pm} = -\frac{(3\gamma+1)\sqrt{3(2\omega+3)}}{4\Gamma_1} \mp \frac{(3\gamma+1)\Gamma_2+4\Gamma_1}{8\Gamma_1} \quad (74)$$

Since, in the limit $\tau \rightarrow \pm\infty$, H_E and T_E are proportional ($T_E = 3H_E$), we can analyse by using T_E in the previous subsection.

Summary:

$$\begin{aligned} a_E(\tau) &\text{ runs from } +\infty \text{ to } 0 \text{ for } \gamma < -\frac{1}{3}. \\ a_E(\tau) &\text{ runs from } 0 \text{ to } +\infty \text{ for } \gamma > -\frac{1}{3}. \end{aligned}$$

2. $\Gamma_1 > 0$ case

From Eq.(58), we can write the scale factor for this case as follows.

$$\begin{aligned} a_E(\tau) &= e^{\alpha_E(\tau)} \\ &= c_2 e^{\frac{(3\gamma+1)\sqrt{3(2\omega+3)}}{4\Gamma_1}c\tau} |\sinh(c\tau)|^{-\frac{[(3\gamma+1)\Gamma_2+4\Gamma_1]}{8\Gamma_1}}. \end{aligned} \quad (75)$$

where $c_2 = e^{\frac{(3\gamma+1)B}{4}}(\frac{q}{c})^{-\frac{[(3\gamma+1)\Gamma_2+4\Gamma_1]}{8\Gamma_1}}$. Since as we saw in the section VII.1.B, $\eta < 1$ and $\Gamma_1 > 0$ are not consistent with each other, we consider the case $\eta > 1$. In this region there is singularity at $\tau = 0$. So we need to consider the limit $\tau \rightarrow 0$:

$$\begin{aligned} a_E(\tau) &\rightarrow |\tau|^{-\eta/3} \\ &= |\tau|^{-\frac{(3\gamma+1)\Gamma_2+4\Gamma_1}{8\Gamma_1}}. \end{aligned} \quad (76)$$

As $\tau \rightarrow 0$, the behavior of the scale factor always goes to positive infinity because $\eta > 1$. As $\tau \rightarrow \pm\infty$, $a_E(\tau)$ can be written as

$$e^{H_{E\pm}\tau}$$

where

$$H_{E\pm} = \frac{(3\gamma+1)\sqrt{3(2\omega+3)}}{4\Gamma_1} \mp \frac{[(3\gamma+1)\Gamma_2+4\Gamma_1]}{8\Gamma_1}. \quad (77)$$

By the same analysis of subsection VII.1.B we can write the solutions. Under the condition $\omega > \omega_{\Gamma_1}$,

$$H_{E-} > 0 \text{ and } H_{E+} < 0.$$

The scale factor evolves from 0 to $+\infty$ for $\tau \in (-\infty, 0)$ and from ∞ to 0 for $\tau \in (0, \infty)$.

Now we study $a_E(t_E)$. First consider in the limit $\tau \rightarrow 0$. In this limit $\Gamma_1 < 0$ case is regular. Therefore we investigate $\Gamma_1 > 0$ case. From Eqs.(66) and (76) we write $a_E(t_E)$ as

$$a_E(t_E) \sim [(1-\eta)\text{sign}(\tau)t_E]^{\frac{\eta}{3(\eta-1)}}. \quad (78)$$

When $\frac{\eta}{3(\eta-1)} > 1$ i.e. $1 < \eta < \frac{3}{2}$, the scale factor will accelerate while when $\frac{\eta}{3(1-\eta)} < 1$ i.e. $\eta > \frac{3}{2}$ the scale factor will decelerate. To accelerate, ω should satisfy

$$\begin{aligned} \omega &> \omega_{\Gamma_2} \text{ for } \gamma < -\frac{1}{3}. \\ \omega &< \omega_{\Gamma_2} \text{ for } \gamma > -\frac{1}{3}. \end{aligned}$$

In Fig.7, region IV correspond to these relations. For deceleration ω should satisfy

$$\omega > \omega_{\Gamma_2} \text{ for } \gamma > -\frac{1}{3}.$$

$$\omega < \omega_{\Gamma_2} \text{ for } \gamma < -\frac{1}{3}.$$

Region III and V satisfy these relations.

In both cases ($\Gamma_1 > 0$ or $\Gamma_1 < 0$), in the limit $\tau \rightarrow \pm\infty$ the scale factor $a_E(t_E)$ behaves as

$$a_E(t_E) \sim t_E^{H_{E\pm}/T_{E\pm}}. \quad (79)$$

Since we already know that T_E and H_E satisfy $T_{E\pm} = 3H_{E\pm}$, from Eq.(79), $a_E(t_E)$ can be written

$$a_E(t_E) \sim t_E^{1/3}.$$

It is interesting to compare with Einstein general relativity where $a_E(t_E)$ behaves as

$$a_E(t_E) \sim t_E^{2/3} \text{ for dust } (\gamma = 0).$$

$$a_E(t_E) \sim t_E^{1/2} \text{ for radiation } (\gamma = 1/3).$$

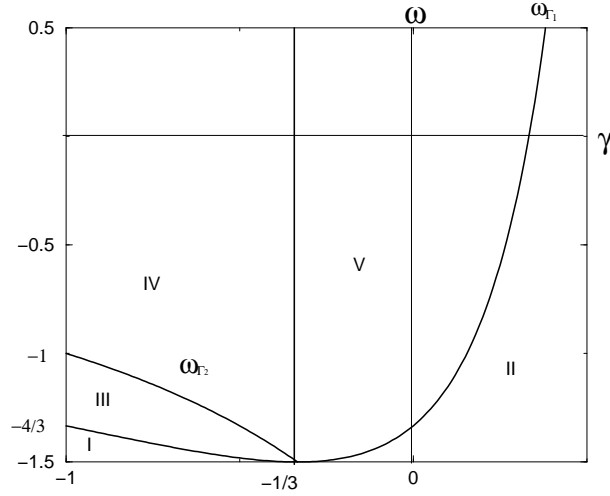


FIG. 7. The classification of phase space in Einstein frame

We summarize the behavior of the scale factor in terms of t_E .

- Region I: $T_{E-}(= 3H_{E-}) < 0, T_{E+}(= 3H_{E+}) < 0$. The universe described by $a_E(t_E)$ evolves from infinite size to zero as time runs from negative infinity to finite final time t_{Ef} .

- Region II: $T_{E-} > 0, T_{E+} > 0$. The universe evolves from zero size to infinite one as time runs from finite initial time t_{Ei} to positive infinity.
- Region III, IV and V: $T_{E-} > 0$ and $T_{E+} < 0$. The universe evolves from zero to infinite size as time runs from finite initial time t_{Ei} to infinity for $\tau \in (-\infty, 0)$ and infinite size to zero as time runs from negative infinity to finite final time t_{Ef} for $\tau \in (0, +\infty)$. Furthermore, in the limit $\tau \rightarrow 0$ the universe has acceleration or deceleration regime which depends on the range of η . Region III and V is decelerationary while region IV is accelerationary phase.

VIII. DISCUSSION AND CONCLUSION

We have considered string motivated Brans-Dicke(BD) cosmology with perfect fluid type matter which arise when a certain kind of the dilaton coupled p-brane gas is dominating the universe. This is the complementary study to our earlier work [7,8], where p-brane gas that does not couple to the dilaton was studied. Cosmology is classified into 16 phases according to the asymptotic behavior of the time interval and the scale factor. This is qualitatively similar to the result obtained before for the dilaton coupled case [7,8]. In string frame, there is a phase where the cosmology has no singularity, namely, region II in Fig 3. In Einstein frame, contrary to the string frame, there is no singularity free phase. This is partly due to the difference of cosmic time(t in string frame and t_E in Einstein frame) and partly due to the dilaton factor relating two frames. In asymptotic regime, $\tau \rightarrow \pm\infty$, the behavior of the scale factor is $t_E^{1/3}$. To our surprise the inflationary regime of the dilaton-graviton string cosmology is gone in the presence of the matter. The matter contribution seems to give mass term or potential to the dilaton regulating the dilaton from growing dilaton kinetic energy [4].

In a recent study [10], with the assumption of holographic principle, it was argued that this principle requires the existence of graceful exit by smoothly connecting the pre and post big-bang branches. According to the [11], all cosmological solutions of p-brane dominating

the universe can be mapped to the present case or the case studied in [7,8]. In any case, there is no solution which exhibit both inflation and graceful exit. Therefore our result draws a negative conclusion to what has been said in [10].

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